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Predicting Liquefaction Displacements with Application to Field Experience

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ABSTRACT

Liquefaction has caused severe distress or failure to buildings, bridges and dams during seismic events. Such damage arises from the soil displacements occurring upon liquefaction. The potential for displacement is a widespread concern because many structures in seismic areas were built when liquefaction and its effects were less understood. Observations of damage from a number of earthquakes, including San Fernando 1971, Loma Prieta 1989, and Kobe 1995, has prompted a re-examination of many of these structures, and they are often being retrofitted at great expense. A key factor in such examinations is the magnitude of the displacements arising from soil liquefaction.

A total stress dynamic approach is presented for estimating displacements from seismically-induced liquefaction. The approach is derived from widely-accepted assumptions for evaluating triggering, flow slide potential, and limited displacements, combining these different evaluations into a single analysis while eliminating some of the inherent simplifications in current procedures. An explicit finite-difference analysis is performed with the earthquake motion applied to the base. Triggering of liquefaction is continuously assessed by weighting each cycle of shear stress. Post-liquefaction stiffness and strength properties are assigned to an element when sufficient cycles of shear stress have accumulated. Elements continue to liquefy and respond to inertia loads as the shaking proceeds, causing the displacements to increase with the duration of shaking. The method is described and then compared with field experience as captured in the Bartlett and Youd (1995) empirical equations.

INTRODUCTION

In assessing liquefaction response there are two concerns: (1) Will liquefaction be triggered in significant zones? and if so, (2) what displacements will occur? If liquefaction is not triggered, the displacements are often small and can be computed using pre-liquefaction material properties.

Liquefaction can cause a large drop in stiffness and strength, and damaging displacements may result if the liquefied zones are of sufficient extent. These displacements can be predicted from two approaches: (1) empirical equations based on past experience; and (2) mechanics-based methods. Empirical relationships are valuable tools since they directly include the intangible and uncertain aspects of actual field response. But they are often limited to specific topographic and material conditions and can be difficult to use for evaluating two-dimensional effects, such as from site remediation. Empirical equations also provide a convenient method of calibrating the mechanics-based approaches.

Mechanics-based methods estimate the field response by approximating soil behaviour using numerical models. The models capture the fundamental physics of dynamic soil response, although their success is limited by inherent simplifications. These methods require some knowledge of soil properties, such as stiffness and strength. The mechanics-based approaches vary in complexity from equivalent-linear total stress methods to advanced effective stress simulations. The accepted state of practice is often a three-phase total stress procedure:

- Triggering Evaluation: Zones of liquefaction are predicted by comparing estimates of applied loading or cyclic shear stress to the anticipated resistance to liquefaction. The applied loading is often approximated using equivalent-linear techniques, such as the computer program SHAKE (Idriss and Sun 1992). The resistance to triggering is typically based on empirical correlations to in-situ tests, such as the standard penetration test (Youd and Idriss, 1998).
- Flow Slide Evaluation: If significant zones of liquefaction are predicted, the potential for a flow slide is evaluated using limit equilibrium techniques. The susceptibility to large deformations is found by assigning residual or post-liquefaction strengths to the liquefied zones and appropriate undrained strengths to the non-liquefied material. If the safety factor is less than or near 1, the structure is not stable.

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- Displacement Evaluation: Large but limited displacements may occur even if the structure has sufficient strength to maintain overall stability. These limited displacements accumulate during the earthquake and are a direct result of the continuing inertia loading. Techniques for estimating these displacements are often based on Newmark (1965), and model the displacing soil as a rigid block translating on a plane.

The current practice has several advantages, including wide experience and the relative simplicity of procedure and input. There are also some drawbacks including the disjointed nature of the procedure, the simplified evaluation of triggering, and the crude modeling of post-liquefaction mechanics. The total stress approach presented below attempts to capitalize on the experience and understanding of the current state of practice, while synthesizing these different components into a more coherent, rational procedure.

DESCRIPTION OF SYNTHESIZED APPROACH

The framework of the synthesized approach is the numerical solution technique. A finite-difference continuum model is used, although the approach can be adapted to other methods such as finite element. General two-dimensional structures are discretized into elements, masses are lumped at nodal points, and constitutive relationships and properties are assigned. The earthquake motion is applied to the base of the model. An explicit solution scheme is used where the dynamic equations of equilibrium are satisfied for each mass at every timestep. This approach maintains dynamic equilibrium throughout the imposed motion, so the effects of material softening, dynamic inertia, gravity loads, and imposed stress changes are directly considered: each element strains and deforms to maintain dynamic equilibrium. This scheme requires very small timesteps, often less than 0.0001 sec, but easily permits non-linear response and large strain behaviour. The commercial finite-difference code FLAC was used to perform the calculations (Itasca, 1998).

The synthesized approach begins with pre-liquefied properties assigned to all elements and the model is brought to static equilibrium. The earthquake history is then applied and the dynamic analysis is performed in the time domain.

Analysis Phase 1 – Pre-Liquefaction

The analysis prior to liquefaction is similar to the equivalent linear method: a linear elastic-plastic constitutive model is used in combination with Rayleigh stiffness-proportional viscous damping. Elastic moduli are estimated from the maximum shear modulus G_{max} and a modulus reduction factor (MRF). However, unlike equivalent linear methods, the synthesized approach is not iterative. Appropriate values of MRF and damping are selected prior to the analysis. Guidance for selecting these values can be obtained from SHAKE analyses, or based on experience. Appropriate strength values can also be assigned.

The pre-liquefied properties are maintained in each element and are not modified unless that element liquefies. For the sake of simplicity, changes in pore pressure prior to liquefaction are not directly considered in the approach but can be approximately included through a reduction in stiffness.

Analysis Phase 2 – Triggering of Liquefaction

The triggering of liquefaction is evaluated by tracking the dynamic shear stress history on the horizontal plane, τ_{xy}, within each element. The cyclic pulse, τ_{cyc}, is computed at every timestep and is defined as the difference between the current shear stress and the static bias, or $\tau_{cyc} = |\tau_{static} - \tau_{xy}|$. The static bias, τ_{static}, is the τ_{xy} which exists prior to dynamic loading. The irregular shear stress history caused by the earthquake is interpreted as a succession of half cycles, with the contribution of each half cycle to triggering determined by its maximum value of τ_{cyc}. This definition of cyclic loading is shown schematically in Figure 1.

A cumulative damage approach is used to combine the effects of each half cycle. This approach converts the non-uniform τ_{cyc} history into an equivalent series of uniform stress cycles. For convenience, the amplitude of this uniform history is set equal to τ_{15}, the value of τ_{cyc} required to cause liquefaction in 15 cycles. This process transforms each half cycle of τ_{cyc} into an equivalent number of cycles (N_{eq}) at τ_{15}. Liquefaction is triggered when the ΣN_{eq} exceeds 15.

This transformation to equivalent cycles is accomplished using the cyclic strength curve as a weighting curve, as shown in Figure 2. Conversion from one-half cycle of τ_{cyc} to N_{eq} cycles at τ_{15} uses the following procedure:

1. Use the weighting curve to find N_{liq}, the number of uniform cycles of τ_{cyc} required to cause liquefaction.
2. Multiply N_{liq} by 2 to find the required number of half cycles to cause liquefaction.
3. Compute the ratio of loading received to loading required for liquefaction: $1 / (2^* N_{liq})$.
4. Compute $N_{eq} = 15 / (2^*N_{liq})$.

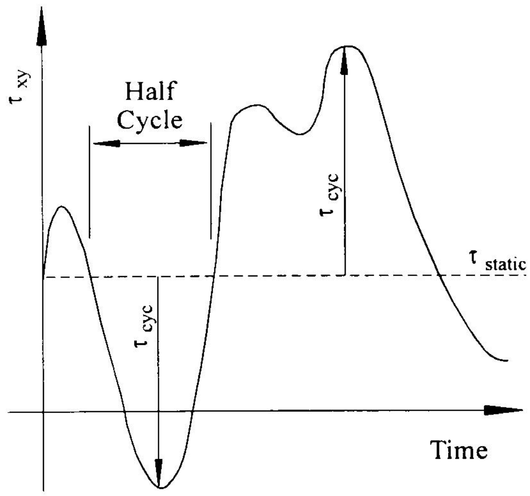

Figure 1 — Example of τ_{cyc} and Half Cycle

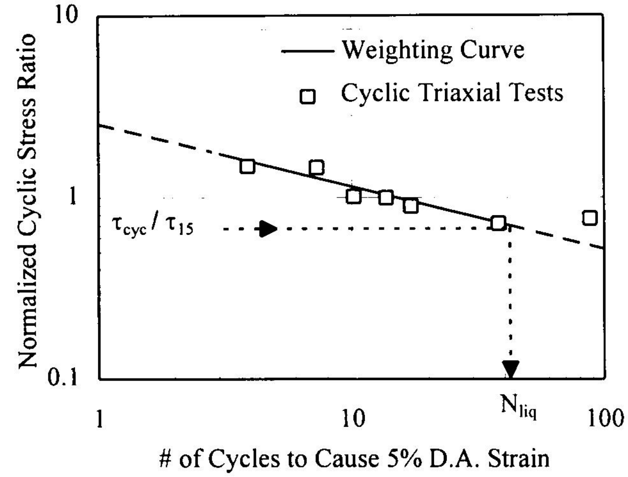

Figure 2 — Weighting Curve

An idealized stress-strain curve showing the onset of liquefaction is given in Figure 3. The synthesized method attempts to capture this behaviour by continuously evaluating τ_{cyc}. Triggering is assumed when the current value of τ_{cyc} is sufficient to bring ΣN_{eq} to 15 at the end of the next half cycle. The magnitude of τ_{xy} is considered as it is often more likely to initiate liquefaction when the cyclic pulse and the static bias are in the same direction.

Since the triggering of each element is evaluated separately, liquefaction first occurs in the most susceptible areas and then spreads outward with further dynamic loading in a realistic manner.

Analysis Phase 3 – Post-Liquefaction Element Behaviour

Several changes are made to model liquefied conditions. The stiffness is reduced and a residual strength is specified at the instant of triggering. The constitutive model is also modified so that unloading increments use a stiffer modulus than loading increments, as shown in Figure 4. This bilinear model permits the accumulation of displacement with each cycle while still maintaining a simple elastic-plastic formulation. Due to the large hysteretic damping component, the viscous damping coefficients in the liquefied elements are also reduced.

Another common feature of liquefied behaviour is the occurrence of 100% pore pressure (r_u = 100%) at stress reversals.

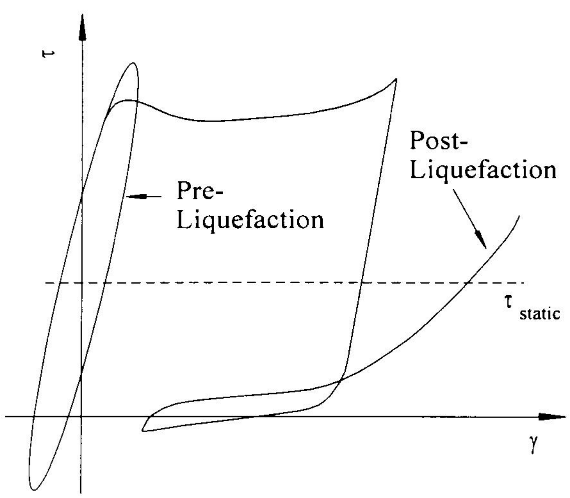

Figure 3 — Idealized Stress-Strain Behaviour
With Liquefaction

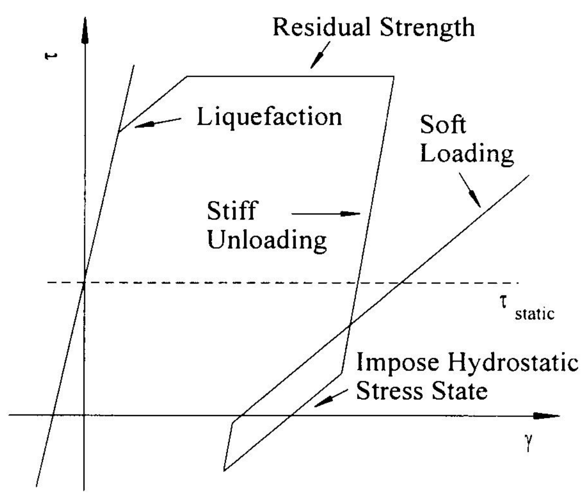

Figure 4 —Stress-Strain Behaviour Modeled
in Synthesized Approach

This is simulated in the analysis by imposing a hydrostatic stress state whenever a liquefied element has a shear stress reversal. The lateral stresses are set equal to the vertical stress and the shear stress is removed: $\sigma_{xx} = \sigma_{yy}$, $\sigma_{zz} = \sigma_{yy}$, and $\tau_{xy} = 0$. Imposing these stresses momentarily removes all shear stress from the element, as is the case with 100% pore pressure. This often causes a force imbalance, and the model will strain until an equilibrium state is again achieved.

Comparison with Laboratory Data

A better understanding of the synthesized model can be obtained by comparing its basic assumptions with an actual laboratory test. Results from a cyclic direct simple shear test on a medium Nevada Sand are shown in Figure 5 (Bardet, 1997). It is difficult to define the onset of liquefaction from this test, as the element appears to transition into liquefied behaviour over about one cycle of loading. This transition starts about a half cycle previous to Point 1, as shown on Figure 5, and continues until about Point 3. For simplicity, the triggering of liquefaction might be assumed to occur at Point 1 as this marks the start of the first strongly dilative response. This is shown in Figure 5b by the increase in effective stress between Point 1 and Point 2, which is accompanied by a soft stress-strain response. Unloading from Point 2 is contractive and stiff, although the behaviour quickly softens as the shear stress reverses direction. The minimum effective stress, Point 3, occurs near the stress reversal. Although r_u does not reach 100% on this transition cycle, it essentially reaches this value on later cycles. Loading from point 3 is dilative and soft, with a typical concave stress-strain curve to Point 4. Unloading is again steep and contractive, and the process of dilation and contraction continues.

While the approach shown in Figure 4 captures much of the behaviour of Figure 5, there are some key differences. The brief transition into liquefied behaviour is not accounted for in the synthesized approach, as liquefied properties are assigned immediately upon triggering. The bilinear model also causes the shear stress to increase relatively quickly after a stress reversal. As a result, the model is most appropriate for situations where strains will accumulate primarily in one direction. Another notable difference is the area of the post-liquefaction stress strain loops. The concave shape of the test data reduces this area, leading to less hysteretic damping than with the bilinear model. The difference should be smaller for elements having a pronounced one-sided response, such as those with a significant static bias.

COMPARISON WITH BARTLETT AND YOUD EMPIRICAL METHOD

The empirical relationship developed by Bartlett and Youd (1995) for sloping ground conditions was selected as a measure of field response for comparison with the synthesized approach. The finite-difference model consisted of a single column of elements with boundary conditions simulating an infinite slope. The geometry and properties of the column were selected to fall within the limits of the empirical procedure. A brief description is given in Table 1.

Input Properties for Synthesized Analysis

Four critical properties are required for the pre-liquefaction phase: shear modulus, bulk modulus, density, and viscous

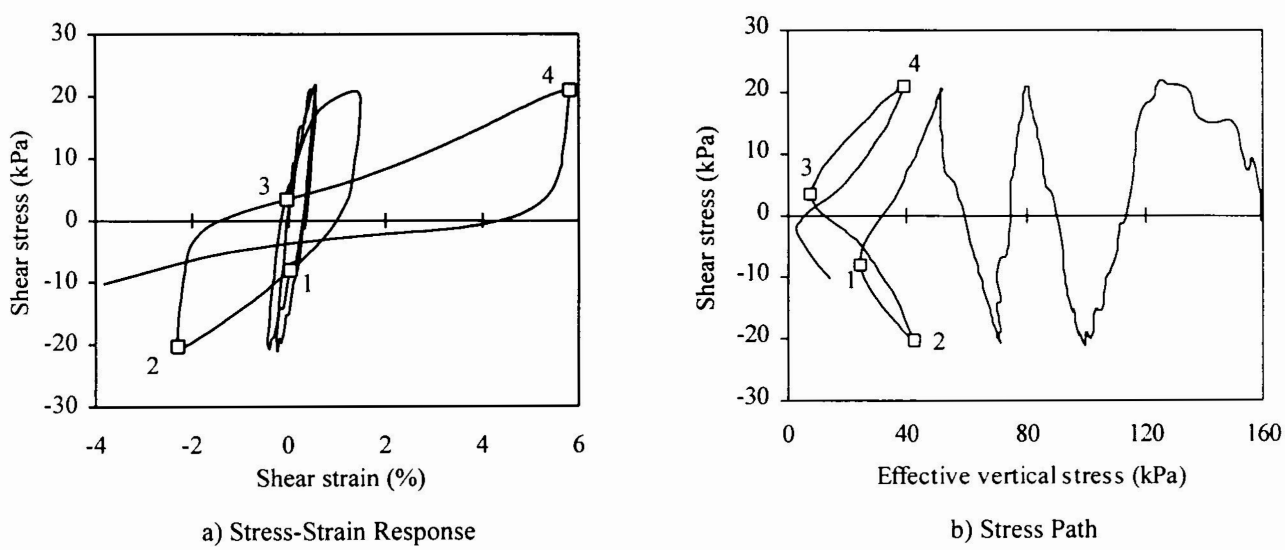

Figure 5 — Cyclic Simple Shear Test Results from VELACS (Bardet, 1997)

Table 1 – Description of Infinite Slope Model

Item	Base Analysis	Range Evaluated	Item	Base Analysis	Range Evaluated
Height of Column	10 m	–	$(N_1)_{60}$	10	7 to 13
# of Elements	10	–	Density	19 kN/m^3	–
Depth to GW	2 m	–	S_r/σ_{vo}'	0.13	0.08 to 0.20
Fines content	–	5%, 45%	γ_r	2%	2% to 5%
D_{50}	–	0.8 mm, 0.1 mm	K_{2max}	$20^*((N_1)_{60})^{1/3}$	–
Slope	4%	2% to 6%	Foundation	Rock	Rock, 40m Alluvium

damping coefficients. G_{max} was estimated from the blowcount and a relationship for K_{2max} proposed by Seed et al. (1986): $K_{2max} = 20^*((N_1)_{60})^{1/3}$. Modulus reduction factors were estimated from a SHAKE analysis of the column, using modulus and damping curves proposed by Idriss (1992). The results from SHAKE were simplified and two modulus reduction factors were selected: one for the bottom 7 metres and one for the top 3 metres. The bulk modulus was assumed equal to G_{max} above the water table, and to $50^*MRF^*G_{max}$ in saturated elements. Damping coefficients corresponding to the MRF values were selected by averaging the SHAKE damping predictions. The center frequency associated with the damping was selected from the frequency content of the shear stress histories predicted by SHAKE. The values for MRF and damping were verified by simulating the SHAKE column with a finite-difference column.

The weighting curve used for this study is shown in Figure 2. The MCEER triggering chart (Youd and Idriss, 1998) was used to estimate τ_{15} from $(N_1)_{60}$. A correction for overburden pressure, K_σ, was included following the recommendations in the MCEER report. K_α, the correction factor for sloping ground, was assumed equal to unity. The magnitude scaling factor, K_m, was not needed as the effect of duration and number of load cycles is directly modeled by the procedure. Damping was reduced in the liquefied elements to 2% of critical to reduce the effect of combining high levels of hysteretic and viscous damping. The post-liquefaction loading modulus, G_{liq}, is defined by the following relation: $G_{liq} = S_r/\gamma_r$, where S_r is the residual strength and γ_r the residual strain. These two parameters are selected to approximate the soft concave stress strain response of post-liquefaction loading. Studies by Dobry indicate values of γ_r as small as 1% to 2% for unidirectional loading (Dobry, 1998), although some test data shows significantly higher values (Byrne et al., 1994). S_r/σ_{vo}' was estimated from laboratory tests with an approximate correlation to field case histories. The unloading modulus was set equal to 10 times the loading modulus, based on a preliminary review of laboratory data.

The input earthquake used for these analysis was the Kagel Canyon record from the Northridge earthquake. The record has the following characteristics, as described by SMDB (1999): $M_w = 6.7$, pga = 0.43g, free-field location on tertiary sandstone. The "within" record determined by the SHAKE analysis was used at the base of the finite-difference column.

Analysis Results

A comparison of displacements from the empirical method and synthesized approach is given in Figures 6 and 7. It is difficult to develop a single estimate from the empirical method due to its sensitivity to input parameters not directly considered by the synthesized approach, primarily D_{50} and fines content (F_{15}). Two combinations of D_{50} and F_{15} were evaluated. D_{50} = 0.8mm and F_{15} = 5% gave 3.9 metres of displacement. This combination represents a clean sand while maintaining a prediction within the bounds of the empirical database: 90% of the observed displacements for sloping ground conditions were less than about 3.8 metres. Reducing D_{50} quickly increases the displacements to well over 10 metres. A second combination, D_{50} = 0.1mm and F_{15} = 45%, gave 1.4 metres of displacement. This combination approximates typical conditions from California as included in the empirical database. This distinction was made to correspond with the California earthquake record used in the analysis. While the properties used in the synthesized approach do not consider such a high fines content, the results should be reasonably appropriate for an uncorrected $(N_1)_{60}$ of about 5 - 6. The average $(N_1)_{60}$ estimated for the liquefied zones from the California observations was 9.5.

Figures 6 and 7 reveal some preliminary but interesting trends. The range shown for each prediction is the effect of earthquake direction, as each column was analyzed with the earthquake oriented in both directions. This effect is significant, with the larger prediction up to 1.7 times the smaller prediction. One direction gave consistently larger results than the reverse direction, so the effect is not random. Simulating 40 metres of alluvium beneath the 10 metre column more than doubled the displacements. This indicates the importance of site conditions and perhaps frequency content of

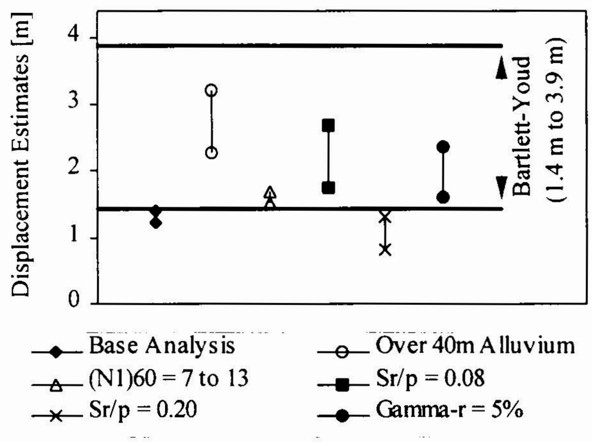

Figure 6 – Comparison of Bartlett-Youd and Synthesized Approach for 4% Slope

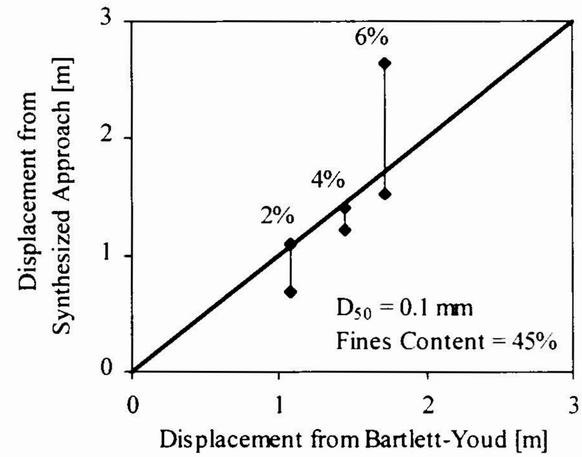

Figure 7 – Comparison of Base Analysis with 2%, 4% and 6% Slopes

the earthquake motion. A non-uniform $(N_1)_{60}$, in this case varying $(N_1)_{60}$ from 13 at the bottom to 7 at the top, made little difference to the displacements. Both residual strength and residual strain are important parameters. The effect of slope embodied in the empirical method seems to be captured by the synthesized approach.

CONCLUSIONS

A total stress model was presented which simulates essential aspects of pre- and post-liquefaction behaviour of sand. The model is applied in an approach that permits simultaneous evaluation of triggering and displacement. A comparison was made to the Bartlett and Youd empirical procedure, and reasonable agreement was obtained. The results indicate the importance of site conditions below the liquefied soil, the direction of loading, and possibly the frequency content of the earthquake to the predicted displacements. These parameters are often not considered in empirical methods.

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